

## WORKSHOP

### GEOMETRIC ANALYSIS IN GEOMETRY AND TOPOLOGY 2017

**Date** : December 19th – 22th, 2017

**Place** : Tokyo University of Science, Morito Memorial Hall

#### Invited speakers

- **Shin Nayatani** (Nagoya Univ., Japan)
- **Shouhei Honda** (Tohoku Univ., Japan)
- **Shinichiroh Matsuo** (Nagoya Univ., Japan)
- **Yoshihiko Matsumoto** (Osaka Univ., Japan)

#### Schedule

.....	10:00–11:00	11:30–12:30	14:00–15:00	15:30–16:30
<b>Dec. 19</b>	Honda-1	Matsumoto-1	Honda-2	Matsumoto-2
<b>Dec. 20</b>	Honda-3	Matsumoto-3	Honda-4	Matsumoto-4
<b>Dec. 21</b>	Matsuo-1	Nayatani-1	Matsuo-2	Nayatani-2
<b>Dec. 22</b>	Matsuo-3	Nayatani-3	Matsuo-4	Nayatani-4

#### Organizers

- Naoyuki Koike (Tokyo University of Science)
- Shu Nakamura (University of Tokyo)
- Mikio Furuta (University of Tokyo)
- Shinichiroh Matsuo (Nagoya University)
- Osamu Kobayashi (former Osaka University)
- Yoshihiko Matsumoto (Osaka University)
- Rafe Mazzeo (Stanford University, Foreign adviser)
- Kazuo Akutagawa (Tokyo Institute of Technology)
- Akiko Takagi (Tokyo Institute of Technology, Secretary)

## Titles & Abstracts

- **Shin Nayatani (Nagoya Univ., Japan)**

### On the Hersch-Yang-Yau inequality

#### and the maximization of the first eigenvalue of a closed surface

In this series of lectures, I will focus on recent progress on metrics that maximize the first eigenvalue of the Laplacian (under area normalization) on a closed surface. In the first lecture, I introduce and prove Hersch-Yang-Yau's inequality (1970, 1980), which was the starting point of the above problem. This is an inequality indicating that the first eigenvalue (precisely, the product of it with the area) is bounded from above by a constant depending only on the genus of the surface. In the second lecture, the recent progress on the existence problem for maximizing metrics will be outlined together with the relation with minimal surfaces in the sphere. In the third lecture, I will discuss Jacobson-Levitin-Nadirashvili-Nigam-Polterovich's conjecture, which explicitly predicts maximizing metrics in the case of genus two, and the affirmative resolution of it (joint work with Toshihiro Shoda). In the fourth lecture I explain the existence theorem for maximizing metrics and the fact that such metrics are given as the induced metrics by minimal immersions into the sphere.

- **Shohei Honda (Tohoku Univ., Japan)**

### Heat flow and Ricci curvature

Heat flow gives a nice approximation of given function. Moreover the regularity properties are related to the lower bound on Ricci curvature of the space. In my talks I will explain them and their recent applications to the study of Gromov-Hausdorff convergence.

- **Sinichiroh Matsuo (Nagoya Univ., Japan)**

### Introduction to gauge theory in higher dimensions

Gauge theory repeats itself, first in four dimension, second in higher dimensions. I will explain higher dimensional gauge theory with central emphasis on its geometric analytic side.

- **Yoshihiko Matsumoto (Osaka Univ., Japan)**

### Foundations and developments of Poincaré-Einstein metrics

Let  $\bar{X}$  be a compact manifold-with-boundary of dimension  $n + 1$ . A Riemannian metric  $g$  on its interior  $X$  is called a Poincaré-Einstein metric (or conformally compact Einstein metric) when the following two conditions are satisfied:

- $\bar{g} = \rho^2 g$  extends nondegenerately up to the boundary, with some specified regularity, where  $\rho \in C^\infty(\bar{X})$  is any fixed boundary defining function;
- $g$  satisfies the Einstein equation  $\text{Ric}(g) = -ng$ .

The fundamental example is the Poincaré metric on the unit ball of  $\mathbb{R}^{n+1}$ .

This talk focuses on the existence problem on Poincaré-Einstein metrics. Topics covered will include the theorem of Graham-Lee (1991) on the possibility of perturbing the Poincaré metric on the unit ball, and results of Anderson (2003) and Gursky-Han (2017) on the failure of general uniqueness and existence.