WORKSHOP

GEOMETRIC ANALYSIS IN GEOMETRY AND TOPOLOGY 2018

Date : March 11th – 14th, 2019
Place : Chuo University (Korakuen Campus) 6th building, 61125

Invited speakers
- **Eric Bahuaud (Seattle University)**
- **Renato Bettiol (City University of New York)**
- **Boris Botvinnik (University of Oregon)**
- **Kei Irie (University of Tokyo)**

Schedule

·····	10:00–11:00	11:30–12:30	14:00–15:00	15:30–16:30
March 11	B. Botvinnik-1	B. Botvinnik-2	R. Bettiol-1	R. Bettiol-2
March 12	E. Bahuaud-1	E. Bahuaud-2	B. Botvinnik-3	B. Botvinnik-4
March 13	K. Irie-1	K. Irie-2	**Short Excursion**	
March 14	R. Bettiol-3	R. Bettiol-4	E. Bahuaud-3	E. Bahuaud-4

Organizers
- Shu Nakamura (University of Tokyo)
- Mikio Furuta (University of Tokyo)
- Shinichiroh Matsuo (Nagoya University)
- Tsuyoshi Kato (Kyoto University)
- Yoshihiko Matsumoto (Osaka University)
- Nobuhiko Otoba (University of Regensburg)
- Rafe Mazzeo (Stanford University, Foreign adviser)
- Kazuo Akutagawa (Chuo University)

Titles & Abstracts of Lectures

- **Eric Bahuaud (Seattle University, USA)**

Lectures 1–2 : Poincaré-Einstein metrics with prescribed conformal infinity

Abstract The seminal work of Fefferman and Graham on conformal invariants introduced an important existence problem: given a compact conformal manifold $(X, [g])$, is it possible to find a manifold with boundary M with boundary$(M) = X$ and a conformally compact Einstein metric g on the interior of M that has $[g]$ as conformal infinity ? In these talks I will introduce this problem and the explain the Graham-Lee theorem that proves the existence of such metrics nearby the hyperbolic metric on the ball. I will then discuss recent work with Lee on low regularity Poincaré-Einstein metrics.

Lectures 3–4 : Ricci flow of asymptotically hyperbolic metrics

Abstract In these two talks I'll survey results concerning the normalized Ricci flow evolving from a conformally compact asymptotically hyperbolic metric. I'll discuss joint work with Mazzeo and Woolgar on the behavior of the renormalized volume along the flow of both asymptotically Poincaré-Einstein metrics and metrics with an even expansion. I'll then discuss joint work with Woolgar on the long-time existence of the flow for rotationally symmetric asymptotically hyperbolic initial data, and ongoing work with Guenther and Isenberg on the stability of these flows.

- **Renato Bettiol (City University of New York, USA)**

Lectures 1– 4 : Applications of Bifurcation Theory to Geometric Analysis

Lecture 1 : Introduction to Bifurcation Theory

Abstract Bifurcation Theory originates from problems in Applied Sciences and Engineering (such as the buckling of columns under compressive stress), and was developed by mathematicians into a powerful toolkit that uses the instability of solutions to certain problems to prove the existence of other solutions nearby. These "bifurcating" solutions are often less symmetric and harder to find directly, but can provide very interesting examples. In this first lecture, I will give a broad overview of bifurcation methods and some classical applications.

Lecture 2 : Bifurcating Constant Mean Curvature Hypersurfaces

Abstract In this second lecture, I will survey on applications of Bifurcation Theory to establish the existence of many hypersurfaces with Constant Mean Curvature. In particular, I will describe a construction of Delaunay-type hypersurfaces in cohomogeneity one manifolds, which generalizes classical Delaunay surfaces.

Lecture 3 : Bifurcating solutions to the classical Yamabe problem

Abstract The third lecture will focus on proving the existence of infinitely many solutions to the (classical) Yamabe problem on certain closed manifolds, that is, Riemannian metrics with constant scalar curvature in prescribed conformal classes. Key examples are products and total spaces of Riemannian submersions.

Lecture 4: Bifurcating solutions to generalizations of the Yamabe problem

Abstract In this last lecture, I will describe how similar bifurcation techniques yield infinitely many solutions to certain generalizations of the Yamabe problem to noncompact manifolds, as well as to higher order problems in conformal geometry, such as the fourth-order Q-curvature problem.

- **Boris Botvinnik (University of Oregon, USA)**

Main topic : Spaces of psc-metrics and moduli spaces of psc-metrics

Lectures 1–3 : The space of positive scalar curvature : constructions, results and conjectures

Abstract Lecture 1 : Geometric constructions and results concerning the relevant spaces of metrics, conformal classes and corresponding moduli spaces. Surgery results, homotopy invariance of the space of psc-metrics under surgery.
Lecture 2 : Dirac operator and psc-curvature, Gromov-Lawson-Stolz results; concordance/isotopy, index theory for families of metrics. Hitchin's results. Index theory and moduli spaces of metrics.
Lecture 3 : Spaces of smooth manifolds and of manifolds with psc-metrics. Review of the results on the moduli spaces following Madse-Weiss, Galatious, Randal-Williams.

Lecture 4 : Positive scalar curvature metrics on manifolds with fibered singularities

Abstract Index-difference map and review of the results following Botvinnik–Ebert–Randal-Williams on the homotopy type of the space of psc-metrics.

- **Kei Irie (University of Tokyo, Japan)**

Lectures 1–2 : Symplectic homology

Abstract Symplectic homology was introduced by Floer-Hofer and Viterbo in early nineties, combining two important ideas in symplectic geometry: symplectic capacity and Floer homology. It is now one of standard tools in symplectic topology and its applications to Hamiltonian dynamics. The goal of my talks will be to outline the theory with a few sample applications, such as a proof of Weinstein conjecture on periodic orbits of Hamiltonian dynamics on symplectic vector spaces, and a proof of Gromov non-squeezing theorem. If time permits, I will discuss a few topics from recent research.